Beyond Mean Field
Confrontation of Different Models with High Transverse Momentum Proton Spectra

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Several models have been proposed to simulate heavy ion reactions beyond the mean field level. The lack of data in phase space regions which may be sensitive to different treatments of fluctuations made it difficult to judge these approaches. The recently published high energy proton spectra, measured in the reaction 94 AMeV Ar + Ta, allow for the first time for a comparison of the models with data. We find that these spectra are reproduced by Quantum Molecular Dynamics (QMD) and Boltzmann Ühling Uhlenbeck (BUU) calculations. Models like Boltzmann Langevin (BL) in which additional fluctuations in momentum space are introduced overpredict the proton yield at very high energies. The BL approach has been successfully used to describe the recently measured very subthreshold kaon production assuming that the fluctuations provide the necessary energy to overcome the threshold in two body collisions. Our new findings suggest that the very subthreshold kaon production cannot be due to two body scattering and thus remains a open problem.

I. INTRODUCTION

The emission of particles of extremely high energies as well as the production of particles at beam energies per nucleon far below the threshold in NN collisions are topics of special interest in the field of heavy ion collisions at intermediate energies. These processes require a strong collectivity of the system or at least a high degree of correlated multiple interactions. An analysis of those processes therefore allows for a study of fluctuations and correlations in the nuclear reactions.

On the theoretical side this is also a very interesting subject because it allows to study the predictions of theoretical simulations beyond mean field level. Mean field calculations have been advanced a long time ago. Time Dependent Hartree Fock calculations allowed to study the kinematics of heavy ion reactions at very low beam energies where two body collisions are negligible [1]. Later, BUU models, like the Boltzmann Ühling Uhlenbeck model (BUU) [2,3], the Vlasov Ühling Uhlenbeck model (VUU) [4], the Boltzmann Nordheim Vlasov (BNV) [5] and the Landau Vlasov model (LV) [6] combined mean field calculations with a collision term. They succeeded to describe several observables at beam energies between 20 AMeV and 1 AGeV.

To go beyond the mean field approach is rather challenging. We are only aware of two approaches which resulted in quantitative predictions of observables. The so-called Boltzmann-Langevin (BL) approach [7–10] relies on a stochastic extension of extended (BUU-like) mean-field theories. Although reasonably well founded from the formal point of view (essentially as well as its progenitor BUU), the application of BL to heavy-ion collisions remains extremely painful, and only approximate methods are presently available in realistic cases [7]. Still, applications of BL have already been proposed for intermediate mass fragment or sub threshold particle production [7]. In the case of kaons, a BL-inspired model has even allowed to provide the order of magnitude of kaon production cross sections around 100 AMeV [11]. The other approach, the so-called Quantum Molecular Dynamics model (QMD) simulates the quantal many-body problem in an approximate (semi classical) way. In this model the kaon production below 100 AMeV is zero and is hence in contradiction with experiment.

Recently, first experimental results on the production of high energy protons in Ar+Ta at 94 AMeV [12] have been reported. These protons, having an energy of several times the beam energy, are presumably produced in

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collisions with a large $\sqrt{s}$ and hence, possibly, in the same type of collisions as the kaons which require a still larger $\sqrt{s}$ value. These data allow now for the first time to study the predictions of the two models in a phase space region where the different approaches to the fluctuations may become relevant. To report about this comparison is the purpose of this letter.

II. QMD AND BL

The QMD model is a n-body theory which is based on a variational principle. It allows to reduce the time evolution of a n - body test wave function to the time evolution of its parameters. In QMD the test wave function is a direct product of n Gaussians. The 6n parameters are the centroids in coordinate and momentum space of the Gaussian wave functions. The time evolution of these centroids are given by Euler Lagrange equations derived by the variation of the Lagrangian. They have the same structure as the classical Hamilton equations. The time evolution of the centroids in momentum space is governed by almost the same potential as employed in BUU or BL approaches. The initial condition is chosen in a way to reproduce the coordinate and momentum space distribution of a cold nucleus. The potential interaction between the nucleons is supplemented by an elastic and an inelastic two body scattering. For details we refer to references [13,14]. The calculations reported here were performed with IQMD, a QMD version which includes isospin explicitly [15]. As far as the single particle spectra are concerned we do not expect a significant difference between the different QMD flavours as long as the NN cross section is not changed.

The Boltzmann Langevin (BL) theory is a one body mean field theory improved by incorporating dynamical fluctuations. A correlation function in momentum space, which can entirely be derived from one body properties and which fulfills the fluctuation dissipation theorem, is employed in each time step of the simulation to spread the trajectory in momentum space around its original value. Out of the distribution of new trajectories one is chosen by a Monte Carlo procedure and propagated to the next time step where the procedure repeats itself. **There are different ways to realize this procedure in numerical simulations [17]. Only one of these propositions has been developed to a simulation program which allows quantitative comparisons with experiments [16].** In this realization one calculates at each time step the actual (fluctuating) value of the quadrupole (octupole) moment of the momentum distribution of nucleons being close together in coordinate space. According to this fluctuating value of the multipole moment and respecting basic energy and momentum conservation laws, one assigns to each of the particles a new momentum with which it is propagated in the next time step. This procedure allows in rare cases

that a large part of the momentum of many fellow nucleons is transferred to a single nucleon. Effectively this corresponds to a many body scattering in which one of the scattering partners may carry a large fraction of the total (local) available momentum. This effective many body scattering is not present in the QMD model. In this respect comparing our BL and QMD simulations allows to disentangle the relevance of collective effects on a given observable.

It should be noted that the above described BL simulations dubbed BL_{PM} (BL realized with the so-called "projection method") are by construction devised to properly account for quadrupole dominated fluctuations. In this respect these simulations do emphasize the collective part (in p-space, not in r-space, where they are local) of fluctuations, at the possible price of over-estimating them if complementing (but overlooked) fluctuation channels do open up. As a consequence, these simulations are expected to be reliable for quadrupole fluctuations or for dynamical situations dominated by quadrupole fluctuations, so that a restriction to the "collective" fluctuations is reasonable. As shown in [17] grid simulations in restricted model space indeed validate quadrupole fluctuations as calculated in the projection method used here (see table 1 of [17]). In turn, differences show up in the high momentum tail of the distribution function, which are, as expected, not necessary accounted for in a multipole-based method of reinjection of fluctuations.

Direct kaon and high energy proton production mechanisms are the same in both (BL_{PM} and QMD) approaches. Whenever two nucleons approach each other closer than $r = \sqrt{\sigma_{tot}/\pi}$ a NN collision takes place. With a probability of $\frac{\sigma_{NN \to X}}{\sigma_{tot}}$ a particle X appears in the exit channel. Thus both, high energy protons as well as kaons, should reflect the probability to have two body collisions with very large $\sqrt{s}$ values as compared to the beam energy. Hence possible differences between the two approaches should have their origin in a different momentum distribution of the incoming particles.

III. RESULTS

A comparison of the models with experimental results is not that easy because we are investigating a part of the phase space which is very dilutely populated. We deal with protons having three times the energy which is available in a first chance NN collision at the same incoming energy per nucleon and which are emitted in sideward direction. This requires an extremely high number of simulations of the reaction.

Figure 1 displays $\frac{d\sigma}{d\Omega dE}$ for four different opening angles in comparison with the experimental results. First of all we observe that, up to proton kinetic energy of 250 MeV, simulations and experiments agree within error bars even in this remote phase space region. Despite
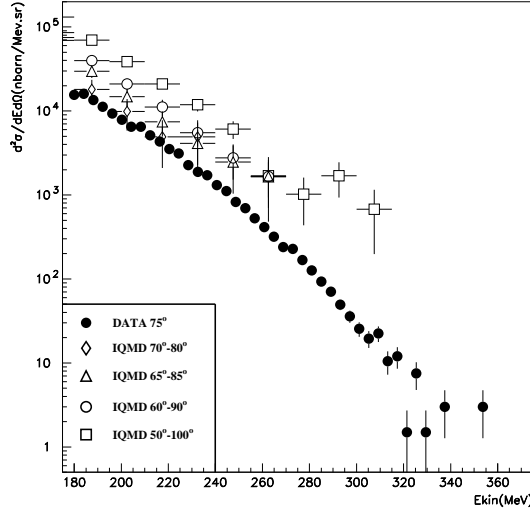
FIG. 1. Comparison of the measured proton spectra for Ar (92AMeV)+Ta [9] to those obtained in QMD using different angular bins.

FIG. 2. Proton spectra for Ar (92 AMeV)+Ta obtained with QMD, BUU and BL_{PM} in comparison to DATA.

of 50,000 simulations of the reaction we are not able to simulate directly the detector acceptance but have to enlarge the acceptance in order to gain statistics: the diamonds correspond to an acceptance of $70° \leq \theta \leq 80°$, the triangles to one of $65° \leq \theta \leq 85°$, the open circles to one of $60° \leq \theta \leq 90°$ and the squares to one of $50° \leq \theta \leq 100°$. We see that between $60°$ and $90°$ the cross section is isotropic in between the error bars and hence $\frac{d\sigma}{d\Omega dE}$ does not change if we enlarge the opening angle. If we enlarge the opening angle even further, the slope stays constant but the absolute value of the cross section increases. By enlarging the opening angle the constant slope value should allow for an extrapolation of the simulated spectra to higher energies where QMD calculations do not allow to explore directly high energy protons.

Figure 2 compares the double differential cross section $\frac{d\sigma}{d\Omega dE}$ with the results of three different transport theories, QMD, BUU and BL_{PM} . This figure displays several interesting features, which need a cautious discussion.

First of all we observe a quite different slope of the spectra of the BL_{PM} calculation on the one side and of the BUU and QMD calculations on the other side. Both slopes are fairly exponential and display an apparent temperature (in between 200-250 MeV) of 76 MeV and 22 MeV, respectively. The difference of the spectra reaches 3 orders of magnitude at proton energies of 225 MeV. A comparison with fig 1 shows that this difference cannot be caused by the enlarged opening angle. We observe as well, that between $E_{kin} = 75$ and $E_{kin} = 200$ MeV, BUU and QMD agree, a fact which has been already discovered a couple of years ago [18]. Below $E_{kin} = 50$ MeV BUU and BL coincide and strongly differ
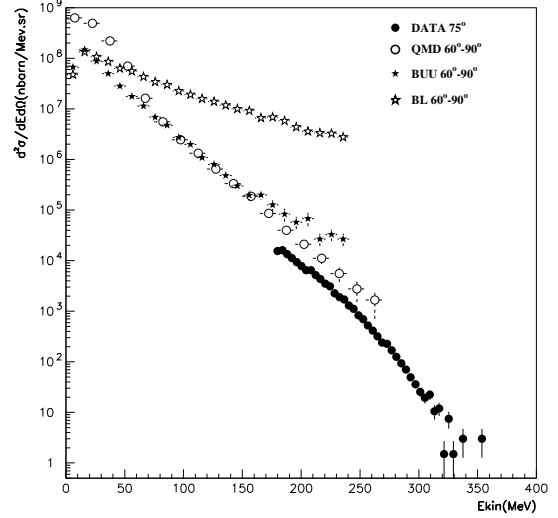
from QMD. This has probably the following reason: For the QMD approach the spectra of all protons has been displayed (to avoid the rather time consuming minimum spanning tree algorithm which defines which nucleons are part of a fragment) whereas in the other two approaches the spectra contains only those protons which are finally not part of a cluster.

The comparison to data is also enlightening. First it should be noted that data are available only in a restricted range of E_{kin} between about 175 and 350 MeV. Other data [19] from reactions induced by 94 AMeV oxygen projectiles show similar proton spectra slopes below 150 MeV. Among the 3 approaches QMD, BUU and BL_{PM} , it is finally QMD which leads, in a small window between 175 MeV and 250 MeV to the best agreement with data (apparent temperature of order 17 MeV) both in absolute values and slope.

A further point, not directly visible in the figure, needs also to be mentioned. BUU and BL_{PM} give the correct time evolution of the one body distribution function of the system if one employs an infinite number of test particles. Therefore the results have been checked on a possible dependence on the number of employed test particles and we show here numbers corresponding to an "asymptotic" regime. It should be noted that for small values of the number of test particles we observed a reduced yield at high kinetic proton energies.

It is remarkable that BUU (a one body mean field approach) and QMD (a n-body molecular dynamics approach) produce in the interesting region the same proton spectra and that both agree with experiment, what confirms the result of ref. [18]. This raises the question whether the additional fluctuations of BL_{PM} (as compared to BUU) can manifest themselves in momentum space. Since BL_{PM} introduces fluctuations which are

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not present in the BUU mean field approach but contained in QMD by construction there seems to be little room for this conjecture. Most probably many body effects manifest themselves as fluctuations (or correlations) in *coordinate space*. These density fluctuations which are washed out in BUU calculations are the origin of the clusterization during a heavy ion collision. Large differences between QMD and BUU have indeed been observed [20] as far as cluster production is concerned.
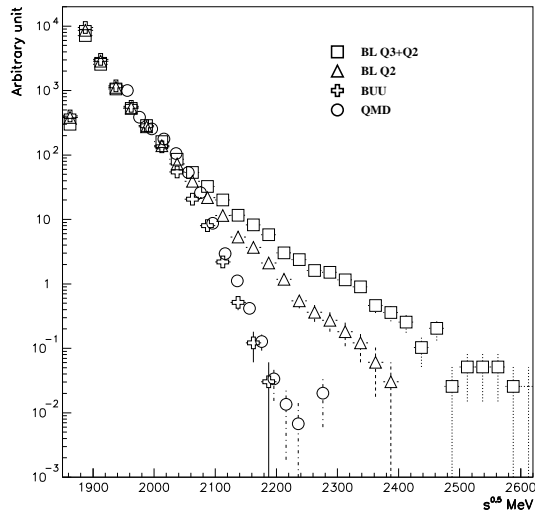

FIG. 3. $s^{1/2}$-distribution for Ar(92 AMeV)+Ta at $b = 0$ fm obtained with QMD, BUU and BL.

As mentioned above, BL_{PM} overestimates the proton yield at high kinetic energies. This strongly enhanced high energy proton component reflects the larger available energies in the NN center of mass system in the BL_{PM} approach, as compared to BUU or QMD. Figure 3 displays the $\sqrt{s}$ distribution of all NN collisions performed during the simulation in the various approaches. We observe a large difference between the models. In QMD and BUU there is no collision with a $\sqrt{s}$ larger than 2.3 GeV, while in BL_{PM} collisions with an energy beyond the threshold for kaon production are observed ($\sqrt{s}_{thres} = 2.548$). The results of BL_{PM} change slightly if one employs octupole moments (Q2+Q3) as the fluctuating quantities as compared to the fluctuations of the quadrupole moments (Q2) only.

Independent of the chosen set up we observe, in BL_{PM} simulations, collisions with much higher $\sqrt{s}$ energies than in the other simulation programs. These energetic collisions produce energetic protons with a too large rate as compared to available data. This probably happens because the Monte Carlo selection employed to choose the momenta out of a given distribution allows that a sizeable fraction of the total momentum is transferred to a single nucleon. This, as already mentioned, mocks up "collective" effects.

Subthreshold kaon production is presently a very active research field [14,21,22] because kaons are considered as a possible messenger from the high density zone of the reaction and hence of a possible onset of the chiral phase transition. Up to recently, however, it was not believed that below 600 AMeV kaons could be observed. Experimentally as well as theoretically the probability to find a kaon becomes exponentially low with decreasing energy and hence the beam or calculation time exceeds present possibilities.

A while ago it has been reported that kaons have been observed in a heavy ion reaction at an energy as low as 92 AMeV [23]. Recently this experiment has been repeated with a comparable result ($\sigma_{K+} = 2.9 \pm 1.6 \cdot 10^{-9} b$ for the reaction Ar + Ta at 92 MeV/N) [24]. It should be noted that the necessary center of mass energy to produce a kaon is 671 MeV + twice the mass of the nucleons. Hence, in the investigated system, $^{36}Ar + ^{48}Ti$ at 92 AMeV, which has a $\sqrt{s} - 84m_p$ of 2.0 GeV, 34% of the total available energy is needed to create a kaon. This points towards a highly collective process. We see that BL_{PM}, in the set up Q2+Q3, reproduces these data. We find 4 pp collision above threshold. In these collisions we produce a kaon with a probability of about $0.25 \cdot 10^{-3}$ [25]. This number has to be divided by $2 \cdot 10^5$ for the number of test particles and the number of events and has to be multiplied by 1.3 b for the total reaction cross section. This yields a kaon production cross section of $1.6 \cdot 10^{-9} b$ which is comparable with the experimental result. This argument should nevertheless be taken with some due caution in view of the very small number of relevant pp collisions. Still it qualitatively provides a coherent picture with the results of [11].

It should be noted that the estimates of [11] rely on a schematic model, built from BL simulations based on quadrupole fluctuations. Only characteristics of the quadrupole do serve as inputs of the model, which can thus be considered as numerically safe (see the above discussions on [17] and internal checks, for example with respect to BUU, as presented in [11]). The model of [11] is thus, by construction, highly collective: it presumably exhausts a sizeable part of the collective source of kaon yield. But it is of course by no means a direct simulation of the BL equation. In this respect, kaon production in QMD, together with the collective schematic model of [11], provide a coherent picture, namely the fact that a dominant fraction of kaon yield does stem from collective effects. In turn, high energy proton production seems to originate from direct (non collective) incoherent two-body processes.

From the above calculations, we thus see that models which describe the proton spectra quantitatively have problems with describing kaon production based on binary collisions. Conversely, models accommodating collective effects, raise problems within explaining high energy proton spectra. This may point to a collective kaon production mechanism not included in the QMD or BUU approach or to strong in medium modification of the properties of strange particles (like a lowering of the

mass) which may lower the threshold for the production. Calculations predict, however, that in nuclear matter the kaon production threshold increases [26]. Therefore it is rather unlikely that modifications of the elementary production process can be the reason for the very sub-threshold kaon production. One could argue that collective effects should also affect the pion production, which is observed [3] to originate from the participant zone. There, however, it is found that the number of observed pion scales well with the number of nn collisions above threshold [3].

IV. CONCLUSIONS

In summary, we can conclude that high energy proton spectra are reproduced by QMD calculations, and to a lesser extent by BUU. Both are not able to reproduce well below threshold kaon production. In turn, BL_{PM} fails to explain high energy proton spectra but either direct [16] or schematic [11] fluctuations reproduce the order of magnitude of the kaon production. **Whether this is only a consequence the realization of the BL equations we use or an inherent problem of the BL approach can only be judged if a different realization is developed. The fact that QMD and BUU have already sufficient fluctuations in momentum space may lead to the conjecture that the Langevin force should generate fluctuations in coordinate space only. How this is possible remains to be seen.** We can conclude that high energy protons do not call for highly collective effects, contrarily to well below threshold kaons.

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